

WINTER – 2018 EXAMINATION

Subject Name: Applied Mathematics

Model Answer

Subject Code:

22201

**Important Instructions to Examiners:**

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		<b>Attempt any FIVE of following:</b>	<b>10</b>
	a)	Define odd and even function with suitable example.	<b>02</b>
	Ans	If $f(-x) = -f(x)$ then the function is an odd function e.g. $f(x) = x^3 + x$ $\therefore f(-x) = (-x)^3 + (-x)$ $= -(x^3 + x)$ $= -f(x)$	$\frac{1}{2}$
		If $f(-x) = f(x)$ then the function is an even function e.g. $f(x) = x^2 + 1$ $\therefore f(-x) = (-x)^2 + 1$ $= x^2 + 1$ $= f(x)$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	b)	If $f(x) = \frac{x^2 + 9}{\sqrt{x-3}}$ , find $f(4) + f(5)$ .	<b>02</b>
Ans	$f(4) + f(5) = \left( \frac{4^2 + 9}{\sqrt{4-3}} \right) + \left( \frac{5^2 + 9}{\sqrt{5-3}} \right)$ $= 25 + \frac{34}{\sqrt{2}} = 49.042$ -	$\frac{1}{2} + \frac{1}{2}$	
	OR	$f(4) = \frac{4^2 + 9}{\sqrt{4-3}} = 25$	$\frac{1}{2}$

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1.	b)	$f(5) = \frac{5^2 + 9}{\sqrt{5-3}} = \frac{34}{\sqrt{2}}$ $\therefore f(4) + f(5)$ $= 25 + \frac{34}{\sqrt{2}}$ $= 49.042$	<p>1/2</p> <p>1</p>
	c)	<p>Find <math>\frac{dy}{dx}</math> if <math>y = (3a)^x + x^{(\log 3)} + x^a + a^a</math></p>	02
	Ans	$\therefore \frac{dy}{dx} = (3a)^x \log 3a + \log 3 \cdot x^{(\log 3)-1} + a \cdot x^{a-1} + 0$ $= (3a)^x \log 3a + \log 3 \cdot x^{(\log 3)-1} + a \cdot x^{a-1}$ <p>OR</p> $\therefore y = (3a)^x + x^{(\log 3)} + x^a + a^a$ $\therefore y = 3^x a^x + x^{(\log 3)} + x^a + a^a$ $\therefore \frac{dy}{dx} = 3^x a^x \log a + a^x 3^x \log 3 + \log 3 \cdot x^{(\log 3)-1} + a \cdot x^{a-1} + 0$ $= 3^x a^x (\log a + \log 3) + \log 3 \cdot x^{(\log 3)-1} + a \cdot x^{a-1}$	<p>1/2+1/2+1/2+1/2</p> <p>1/2+1/2+1/2+1/2</p>
	d)	<p>Evaluate <math>\int x^2 \cdot \log x dx</math></p>	02
Ans	$\int x^2 \cdot \log x dx = \log x \int x^2 dx - \int \left[ \int x^2 dx \cdot \frac{d}{dx} \log x \right] dx$ $= \log x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx$ $= \log x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 dx$ $= \log x \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3} + c$ $= \log x \cdot \frac{x^3}{3} - \frac{x^3}{9} + c$	<p>1/2</p> <p>1/2</p> <p>1</p>	

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1.	e)	Evaluate $\int \frac{dx}{x^2 + 4x + 5}$	02
	Ans	$\int \frac{dx}{x^2 + 4x + 5}$ $\text{Third term} = \frac{(4x)^2}{4 \times x^2} = 4$ $= \int \frac{dx}{x^2 + 4x + 4 - 4 + 5}$ $= \int \frac{dx}{(x+2)^2 + 1}$ $= \frac{1}{1} \tan^{-1} \left( \frac{x+2}{1} \right) + c$ $= \tan^{-1}(x+2) + c$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
	f)	Find the area bounded by the curve $y = \sin x$ , $x$ -axis and the ordinate $x = 0, x = \frac{\pi}{2}$	02
	Ans	$\text{Area } A = \int_a^b y dx$ $= \int_0^{\frac{\pi}{2}} \sin x dx$ $= [-\cos x]_0^{\frac{\pi}{2}}$ $= -[0 - 1]$ $= 1$	<p>1</p> <p>1/2</p> <p>1/2</p>
	g)	State the trapezoidal rule of numerical integration.	02
	Ans	<p>Trapezoidal rule</p> $\int_a^b f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$ <p>where <math>h = \frac{b-a}{n}</math></p>	2
2.		<b>Attempt any THREE of the following:</b>	12
	a)	Find $\frac{dy}{dx}$ if $x^2 + y^2 + xy - y = 0$ at (1, 2)	04

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2.	a)Ans	$\because x^2 + y^2 + xy - y = 0$ $\therefore 2x + 2y \frac{dy}{dx} + x \frac{dy}{dx} + y - \frac{dy}{dx} = 0$ $\therefore 2x + y + (2y + x - 1) \frac{dy}{dx} = 0$ $\therefore (2y + x - 1) \frac{dy}{dx} = -(2x + y)$ $\therefore \frac{dy}{dx} = \frac{-(2x + y)}{2y + x - 1}$ at (1, 2) $\frac{dy}{dx} = \frac{-(2(1) + 2)}{2(2) + 1 - 1} = -1$	<p>1</p> <p>1</p> <p>1</p> <p>1</p>
	b)	If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$ , find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$	04
	Ans	$\because x = a(\cos t + t \sin t)$ $\therefore \frac{dx}{dt} = a(-\sin t + t \cos t + \sin t)$ $= at \cos t$ $\because y = a(\sin t - t \cos t)$ $\therefore \frac{dy}{dt} = a(\cos t + t \sin t - \cos t)$ $= at \sin t$ $\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $\therefore \frac{dy}{dx} = \frac{at \sin t}{at \cos t}$ $= \tan t$ at $t = \frac{\pi}{4}$ $\frac{dy}{dx} = \tan \frac{\pi}{4}$ $= 1$	<p>1</p> <p>1</p> <p>1</p>

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2.	c)	<p>The rate of working of an engine is given by the expression <math>10V + \frac{4000}{V}</math>, where 'V' is the speed of the engine. Find the speed at which the rate of working is the least.</p> <p>Ans The rate of working is, <math>W = 10V + \frac{4000}{V}</math></p> $\therefore \frac{dW}{dV} = 10 - \frac{4000}{V^2}$ $\therefore \frac{d^2W}{dV^2} = \frac{8000}{V^3}$ <p>Consider <math>\frac{dW}{dV} = 0</math></p> $\therefore 10 - \frac{4000}{V^2} = 0$ $\therefore 10 = \frac{4000}{V^2}$ $\therefore V^2 = 400$ $\therefore V = 20, -20$ <p>at <math>V = 20</math></p> $\therefore \frac{d^2W}{dV^2} = \frac{8000}{(20)^3} = 1 > 0$ <p><math>\therefore</math> The speed is <math>V = 20</math> at which the rate of working is least</p>	<p><b>04</b></p> <p>½</p> <p>½</p> <p>½</p> <p>1</p> <p>1</p> <p>½</p>
	d)	<p>A telegraph wire hangs in the form of a curve <math>y = a \cdot \log \left[ \sec \left( \frac{x}{a} \right) \right]</math>. Where 'a' is constant. Show that the curvature at any point is <math>\frac{1}{a} \cos \left( \frac{x}{a} \right)</math>.</p> <p>Ans <math>y = a \log \left( \sec \left( \frac{x}{a} \right) \right)</math></p> $\therefore \frac{dy}{dx} = a \frac{1}{\sec \left( \frac{x}{a} \right)} \sec \left( \frac{x}{a} \right) \tan \left( \frac{x}{a} \right) \left( \frac{1}{a} \right)$ $\therefore \frac{dy}{dx} = \tan \left( \frac{x}{a} \right)$ $\therefore \frac{d^2y}{dx^2} = \sec^2 \left( \frac{x}{a} \right) \left( \frac{1}{a} \right)$ $\therefore \text{Radius of curvature is } \rho = \frac{\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	<p><b>04</b></p> <p>1</p> <p>1</p>

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3.	a)	$\therefore$ Point is $\left(\frac{1}{2}, \frac{1}{2}\right)$ $\therefore$ Equation of tangent is, $y - \frac{1}{2} = -1\left(x - \frac{1}{2}\right)$ $\therefore y - \frac{1}{2} = -x + \frac{1}{2}$ $\therefore x + y - 1 = 0$	1
	b)	Find $\frac{dy}{dx}$ if $y = x^x + x^{\sqrt{x}}$ ----- Ans Let $u = x^x$ $\therefore \log u = \log x^x$ $= x \log x$ $\therefore \frac{1}{u} \frac{du}{dx} = x \cdot \frac{1}{x} + \log x$ $= 1 + \log x$ $\therefore \frac{du}{dx} = u(1 + \log x)$ $= x^x(1 + \log x)$ Let $v = x^{\sqrt{x}}$ $\therefore \log v = \log x^{\sqrt{x}}$ $= \sqrt{x} \log x$ $\therefore \frac{1}{v} \frac{dv}{dx} = \sqrt{x} \cdot \frac{1}{x} + \log x \cdot \frac{1}{2\sqrt{x}}$ $= \frac{1}{\sqrt{x}} + \frac{\log x}{2\sqrt{x}}$ $\therefore \frac{dv}{dx} = v \left( \frac{1}{\sqrt{x}} + \frac{\log x}{2\sqrt{x}} \right)$ $= x^{\sqrt{x}} \left( \frac{1}{\sqrt{x}} + \frac{\log x}{2\sqrt{x}} \right)$ $\therefore y = u + v$ $\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $= x^x(1 + \log x) + x^{\sqrt{x}} \left( \frac{1}{\sqrt{x}} + \frac{\log x}{2\sqrt{x}} \right)$	04
			1/2
			1
			1/2
			1
			1









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4.	b)	$= -2 \int \frac{1}{3t^2 - 4t + \frac{4}{3} - \frac{4}{3} - 3} dt$ $= -2 \int \frac{1}{\left(\sqrt{3}t - \frac{2}{\sqrt{3}}\right)^2 - \left(\frac{\sqrt{13}}{\sqrt{3}}\right)^2} dt$ $= -2 \cdot \frac{1}{2\sqrt{\frac{13}{3}}} \log \left( \frac{\sqrt{3}t - \frac{2}{\sqrt{3}} - \frac{\sqrt{13}}{\sqrt{3}}}{\sqrt{3}t - \frac{2}{\sqrt{3}} + \frac{\sqrt{13}}{\sqrt{3}}} \right) \cdot \frac{1}{\sqrt{3}} + c$ $= \frac{-1}{\sqrt{13}} \log \left( \frac{\sqrt{3} \tan \frac{x}{2} - \frac{2}{\sqrt{3}} - \frac{\sqrt{13}}{\sqrt{3}}}{\sqrt{3} \tan \frac{x}{2} - \frac{2}{\sqrt{3}} + \frac{\sqrt{13}}{\sqrt{3}}} \right) + c$ $= \frac{-1}{\sqrt{13}} \log \left( \frac{3 \tan \frac{x}{2} - 2 - \sqrt{13}}{3 \tan \frac{x}{2} - 2 + \sqrt{13}} \right) + c$ <p>OR</p> <p>Let <math>\tan \frac{x}{2} = t</math></p> $\therefore \sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}, dx = \frac{2dt}{1+t^2}$ $= \int \frac{1}{2\left(\frac{2t}{1+t^2}\right) + 3\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}$ $= \int \frac{1}{4t + 3 - 3t^2} \cdot 2dt$ $= \int \frac{1}{-3\left(t^2 - \frac{4}{3}t - 1\right)} \cdot 2dt$ $= \frac{-2}{3} \int \frac{1}{t^2 - \frac{4}{3}t - 1} dt$ $\text{Third term} = \frac{\left(\frac{-4}{3}t\right)^2}{4t^2} = \frac{4}{9}$	<p>1/2</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p>

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4.	b)	$= \frac{-2}{3} \int \frac{1}{t^2 - \frac{4}{3}t + \frac{4}{9} - \frac{4}{9} - 1} dt$ $= \frac{-2}{3} \int \frac{1}{\left(t - \frac{2}{3}\right)^2 - \left(\frac{\sqrt{13}}{3}\right)^2} dt$ $= \frac{-2}{3} \cdot \frac{1}{2 \cdot \frac{\sqrt{13}}{3}} \log \left( \frac{t - \frac{2}{3} - \frac{\sqrt{13}}{3}}{t - \frac{2}{3} + \frac{\sqrt{13}}{3}} \right) + c$ $= \frac{-1}{\sqrt{13}} \log \left( \frac{\tan \frac{x}{2} - \frac{2}{3} - \frac{\sqrt{13}}{3}}{\tan \frac{x}{2} - \frac{2}{3} + \frac{\sqrt{13}}{3}} \right) + c$ $= \frac{-1}{\sqrt{13}} \log \left( \frac{3 \tan \frac{x}{2} - 2 - \sqrt{13}}{3 \tan \frac{x}{2} - 2 + \sqrt{13}} \right) + c$	<p>1/2</p> <p>1</p> <p>1/2</p>
	c)	<p>Evaluate : <math>\int \sec^3 x \, dx</math></p> <p>Ans Let <math>I = \int \sec^3 x \, dx</math></p> $= \int \sec^2 x \cdot \sec x \, dx$ $= \sec x \int \sec^2 x \, dx - \int \left[ \int \sec^2 x \, dx \cdot \frac{d}{dx} \sec x \right] dx$ $= \sec x \tan x - \int [\tan x \cdot \sec x \cdot \tan x] dx$ $= \sec x \tan x - \int \tan^2 x \cdot \sec x \, dx$ $= \sec x \tan x - \int (\sec^2 x - 1) \cdot \sec x \, dx$ $= \sec x \tan x - \int (\sec^3 x - \sec x) \, dx$ $= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$ $I = \sec x \tan x - I + \log(\sec x + \tan x) + c$ $\therefore 2I = \sec x \tan x + \log(\sec x + \tan x) + c$ $\therefore I = \frac{1}{2} (\sec x \tan x + \log(\sec x + \tan x)) + c$	<p>04</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p>
	d)	<p>Evaluate <math>\int \frac{2x^2 + 5}{(x-1)(x+2)(x+3)} \, dx</math></p>	04

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4.	d)Ans	$\int \frac{2x^2 + 5}{(x-1)(x+2)(x+3)} dx$ <p>Consider <math>\frac{2x^2 + 5}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3}</math></p> $\therefore 2x^2 + 5 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2)$ <p>Put <math>x = 1 \Rightarrow</math>  <math>2(1)^2 + 5 = A(1+2)(1+3)</math>  <math>\therefore A = \frac{7}{12}</math></p> <p>Put <math>x = -2 \Rightarrow</math>  <math>2(-2)^2 + 5 = B(-2-1)(-2+3)</math>  <math>\therefore B = \frac{-13}{3}</math></p> <p>Put <math>x = -3 \Rightarrow</math>  <math>2(-3)^2 + 5 = C(-3-1)(-3+2)</math>  <math>\therefore C = \frac{23}{4}</math></p> $\therefore \frac{2x^2 + 5}{(x-1)(x+2)(x+3)} = \frac{7}{12} \frac{1}{x-1} + \frac{-13}{3} \frac{1}{x+2} + \frac{23}{4} \frac{1}{x+3}$ $\therefore \int \frac{2x^2 + 5}{(x-1)(x+2)(x+3)} dx = \int \left( \frac{7}{12} \frac{1}{x-1} + \frac{-13}{3} \frac{1}{x+2} + \frac{23}{4} \frac{1}{x+3} \right) dx$ $= \frac{7}{12} \log(x-1) - \frac{13}{3} \log(x+2) + \frac{23}{4} \log(x+3) + c$	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2} + \frac{1}{2} + \frac{1}{2}</math></p>

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4.	e)	Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\cot x}} dx$	<b>04</b>
	Ans	$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\cot x}} dx$ $= \int_0^{\frac{\pi}{2}} \frac{1}{1 + \sqrt{\frac{\cos x}{\sin x}}} dx$ $\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \quad \text{----- (1)}$ $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx$ $\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \quad \text{----- (2)}$ <p>Add (1) and (2)</p> $I + I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$ $2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ $2I = \int_0^{\frac{\pi}{2}} 1 dx$ $2I = [x]_0^{\frac{\pi}{2}}$ $2I = \frac{\pi}{2} - 0$ $I = \frac{\pi}{4}$	<p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>

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5.	a)	<b>Attempt any TWO of the following:</b>	<b>12</b>
	Ans	Find area of the region by the parabolas, $y^2 = 9x$ and $x^2 = 9y$	<b>06</b>
		$y^2 = 9x$ ----- (1)	
		$x^2 = 9y$	
		$\therefore y = \frac{x^2}{9}$	
		$\therefore$ equ. (1) $\Rightarrow \left(\frac{x^2}{9}\right)^2 = 9x$	
		$\therefore \frac{x^4}{81} = 9x$	
		$\therefore x^4 = 729x$	
		$\therefore x^4 - 729x = 0$	
		$\therefore x(x^3 - 9^3) = 0$	1
	$\therefore x = 0, 9$		
	Area $A = \int_a^b (y_1 - y_2) dx$		
	$\therefore A = \int_0^9 \left(3\sqrt{x} - \frac{x^2}{9}\right) dx$	1	
	$\therefore A = \left(\frac{3x^{3/2}}{\frac{3}{2}} - \frac{x^3}{27}\right)_0^9$	2	
	$= \left(\frac{3(9)^{3/2}}{\frac{3}{2}} - \frac{9^3}{27}\right) - 0$	1	
	$\therefore A = 27$	1	
	b)	<b>Attempt the following:</b>	<b>06</b>
	(i)	Form a differential equation by eliminating arbitrary constant. If $y = A \sin x + B \cos x$	<b>03</b>
	Ans	$y = A \sin x + B \cos x$	
		$\therefore \frac{dy}{dx} = A \cos x - B \sin x$	1
		$\therefore \frac{d^2y}{dx^2} = A(-\sin x) - B \cos x$	1
		$= -(A \sin x + B \cos x) = -y$	
		$\therefore \frac{d^2y}{dx^2} + y = 0$	1

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<b>5.</b>	b)(ii)	<p>Solve <math>(1+x^3)dy - x^2ydx = 0</math></p> <p>Ans <math>\therefore (1+x^3)dy - x^2ydx = 0</math></p> <p><math>\therefore (1+x^3)dy = x^2ydx</math></p> <p><math>\therefore \frac{dy}{y} = \frac{x^2dx}{1+x^3}</math></p> <p><math>\therefore</math> Solution is,</p> <p><math>\int \frac{dy}{y} = \int \frac{x^2}{1+x^3} dx</math></p> <p style="padding-left: 40px;"><math>= \frac{1}{3} \int \frac{3x^2}{1+x^3} dx</math></p> <p><math>\therefore \log y = \frac{1}{3} \log(1+x^3) + c</math></p>	<b>03</b>
	c)	<p>An electrical circuit containing an inductance <math>L</math> henries resistance <math>R</math> in series with an electromotive force <math>E \sin \omega t</math> satisfies the equation <math>L \frac{di}{dt} + Ri = E \sin \omega t</math>.</p> <p>Find the value of the current at any time <math>t</math>, if initially there is no current.</p> <p>Ans <math>L \frac{di}{dt} + Ri = E \sin \omega t</math></p> <p><math>\therefore \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \sin \omega t</math></p> <p>Comparing with <math>\frac{dy}{dx} + Py = Q</math></p> <p><math>\therefore P = \frac{R}{L}</math> and <math>Q = \frac{E}{L} \sin \omega t</math></p> <p><math>IF = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}</math></p> <p><math>\therefore</math> Solution is</p> <p><math>i \cdot IF = \int Q \cdot IF dt + c</math></p> <p><math>i \cdot e^{\frac{R}{L}t} = \int \frac{E}{L} \sin \omega t e^{\frac{R}{L}t} dt + c</math></p> <p style="padding-left: 40px;"><math>= \frac{E}{L} \int \sin \omega t e^{\frac{R}{L}t} dt + c \text{ -----(1)}</math></p> <p>Let <math>I = \int \sin \omega t e^{\frac{R}{L}t} dt</math></p> <p><math>= \sin \omega t \int e^{\frac{R}{L}t} dt - \int \left[ \int e^{\frac{R}{L}t} dt \cdot \frac{d}{dt} \sin \omega t \right] dt</math></p>	<b>06</b>



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5.	c)	$= \sin \omega t \cdot \frac{e^{\frac{R}{L}t}}{R} - \int \frac{e^{\frac{R}{L}t}}{L} \cdot \cos \omega t \cdot \omega dt$ $= \frac{L}{R} \sin \omega t e^{\frac{R}{L}t} - \frac{L\omega}{R} \int e^{\frac{R}{L}t} \cos \omega t dt$ $= \frac{L}{R} \sin \omega t e^{\frac{R}{L}t} - \frac{L\omega}{R} \left[ \cos \omega t \cdot \int e^{\frac{R}{L}t} dt - \int \left[ \int e^{\frac{R}{L}t} dt \cdot \frac{d}{dt} \cos \omega t \right] dt \right]$ $= \frac{L}{R} \sin \omega t e^{\frac{R}{L}t} - \frac{L\omega}{R} \left[ \cos \omega t \cdot \frac{e^{\frac{R}{L}t}}{L} - \int \left[ \frac{e^{\frac{R}{L}t}}{L} \cdot (-\sin \omega t) \omega \right] dt \right]$ $I = \frac{L}{R} \sin \omega t e^{\frac{R}{L}t} - \frac{\omega L^2}{R^2} \cos \omega t e^{\frac{R}{L}t} - \frac{\omega^2 L^2}{R^2} \int e^{\frac{R}{L}t} \sin \omega t dt$ $I = \frac{L}{R} \sin \omega t e^{\frac{R}{L}t} - \frac{\omega L^2}{R^2} \cos \omega t e^{\frac{R}{L}t} - \frac{\omega^2 L^2}{R^2} I$ $\left( 1 + \frac{\omega^2 L^2}{R^2} \right) I = \frac{L}{R} \sin \omega t e^{\frac{R}{L}t} - \frac{\omega L^2}{R^2} \cos \omega t e^{\frac{R}{L}t}$ $I = \frac{R^2}{R^2 + \omega^2 L^2} \left[ \frac{L}{R} \sin \omega t e^{\frac{R}{L}t} - \frac{\omega L^2}{R^2} \cos \omega t e^{\frac{R}{L}t} \right]$ <p>∴ equation (1) becomes</p> $ie^{\frac{R}{L}t} = \frac{E}{L} \frac{R^2}{R^2 + \omega^2 L^2} \left[ \frac{L}{R} \sin \omega t e^{\frac{R}{L}t} - \frac{\omega L^2}{R^2} \cos \omega t e^{\frac{R}{L}t} \right] + c$ <p>initially <math>i = 0</math></p> <p>∴ when <math>t = 0, i = 0</math></p> $\therefore 0 = \frac{E}{L} \frac{R^2}{R^2 + \omega^2 L^2} \left[ \frac{-\omega L^2}{R^2} \right] + c$ $\therefore 0 = \frac{-E\omega L}{R^2 + \omega^2 L^2} + c$ $\therefore c = \frac{E\omega L}{R^2 + \omega^2 L^2}$ $\therefore ie^{\frac{R}{L}t} = \frac{E}{L} \frac{R^2}{R^2 + \omega^2 L^2} \left[ \frac{L}{R} \sin \omega t e^{\frac{R}{L}t} - \frac{\omega L^2}{R^2} \cos \omega t e^{\frac{R}{L}t} \right] + \frac{E\omega L}{R^2 + \omega^2 L^2}$ $\therefore i = \frac{ER}{R^2 + \omega^2 L^2} \left[ \sin \omega t - \frac{\omega L}{R} \cos \omega t \right] + \frac{E\omega L}{R^2 + \omega^2 L^2} e^{-\frac{R}{L}t}$	<p>1/2</p> <p>1</p> <p>1</p> <p>1</p>

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5.	c)	<p>OR</p> $L \frac{di}{dt} + Ri = E \sin \omega t$ $\therefore \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \sin \omega t$ <p>Comparing with <math>\frac{dy}{dx} + Py = Q</math></p> $\therefore P = \frac{R}{L} \text{ and } Q = \frac{E}{L} \sin \omega t$ $IF = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}$ <p><math>\therefore</math> Solution is</p> $i \cdot IF = \int Q \cdot IF dt + c$ $i \cdot e^{\frac{R}{L}t} = \int \frac{E}{L} \sin \omega t e^{\frac{R}{L}t} dt + c$ $= \frac{E}{L} \int \sin \omega t e^{\frac{R}{L}t} dt + c \text{ ----- (1)}$ $\left[ \therefore \int e^{ax} \sin bxdx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) \right]$ $\therefore \int e^{\frac{R}{L}t} \sin \omega t dt = \frac{e^{\frac{R}{L}t}}{\frac{R^2}{L^2} + \omega^2} \left[ \frac{R}{L} \sin \omega t - \omega \cos \omega t \right]$ <p><math>\therefore</math> equation (1) becomes</p> $i \cdot e^{\frac{R}{L}t} = \frac{E}{L} \frac{e^{\frac{R}{L}t}}{\frac{R^2}{L^2} + \omega^2} \left[ \frac{R}{L} \sin \omega t - \omega \cos \omega t \right] + c$ $= \frac{ELe^{\frac{R}{L}t}}{R^2 + \omega^2 L^2} \left[ \frac{R}{L} \sin \omega t - \omega \cos \omega t \right] + c$ <p>initially <math>i = 0, \therefore</math> when <math>t = 0, i = 0</math></p> $\therefore 0 = \frac{EL}{R^2 + \omega^2 L^2} [-\omega] + c$ $\therefore c = \frac{\omega EL}{R^2 + \omega^2 L^2}$ $\therefore i \cdot e^{\frac{R}{L}t} = \frac{ELe^{\frac{R}{L}t}}{R^2 + \omega^2 L^2} \left[ \frac{R}{L} \sin \omega t - \omega \cos \omega t \right] + \frac{\omega EL}{R^2 + \omega^2 L^2}$ $\therefore i = \frac{EL}{R^2 + \omega^2 L^2} \left[ \frac{R}{L} \sin \omega t - \omega \cos \omega t \right] + \frac{\omega EL}{R^2 + \omega^2 L^2} e^{-\frac{R}{L}t}$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>

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Contact - 9136008228

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<b>5.</b>	c)	<p>OR</p> $L \frac{di}{dt} + Ri = E \sin \omega t$ $\therefore \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L} \sin \omega t$ <p>Comparing with <math>\frac{dy}{dx} + Py = Q</math></p> $\therefore P = \frac{R}{L} \text{ and } Q = \frac{E}{L} \sin \omega t$ $IF = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}$ <p><math>\therefore</math> Solution is</p> $i \cdot IF = \int Q \cdot IF dt + c$ $i \cdot e^{\frac{R}{L}t} = \int \frac{E}{L} \sin \omega t e^{\frac{R}{L}t} dt + c$ $= \frac{E}{L} \int \sin \omega t e^{\frac{R}{L}t} dt + c \text{ ----- (1)}$ $\left[ \because \int e^{ax} \sin bxdx = \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin \left( bx - \tan^{-1} \left( \frac{b}{a} \right) \right) \right]$ <p><math>\therefore</math> equation (1) becomes,</p> $i \cdot e^{\frac{R}{L}t} = \frac{E}{L} \frac{e^{\frac{R}{L}t}}{\sqrt{\frac{R^2}{L^2} + \omega^2}} \sin \left( \omega t - \tan^{-1} \left( \frac{\omega}{\frac{R}{L}} \right) \right) + c$ $i = \frac{E}{\sqrt{R^2 + L^2 \omega^2}} \sin \left( \omega t - \tan^{-1} \left( \frac{\omega L}{R} \right) \right) + ce^{-\frac{R}{L}t}$ <p>initially <math>i = 0</math></p> <p><math>\therefore</math> when <math>t = 0, i = 0</math></p> $\therefore 0 = \frac{E}{\sqrt{R^2 + L^2 \omega^2}} \sin \left( -\tan^{-1} \left( \frac{\omega L}{R} \right) \right) + c$ $\therefore c = \frac{-E}{\sqrt{R^2 + L^2 \omega^2}} \sin \left( -\tan^{-1} \left( \frac{\omega L}{R} \right) \right)$ $\therefore i = \frac{E}{\sqrt{R^2 + L^2 \omega^2}} \left( \sin \left( \omega t - \tan^{-1} \left( \frac{\omega L}{R} \right) \right) - \sin \left( -\tan^{-1} \left( \frac{\omega L}{R} \right) \right) e^{-\frac{R}{L}t} \right)$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>

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6.		<b>Attempt any TWO of the following:</b>	<b>12</b>															
	a)(i)	Using trapezoidal rule, evaluate the approximate value of $\int_0^4 \sqrt{x} dx$ , given by <table border="1" style="margin: 10px auto;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>y = <math>\sqrt{x}</math></td> <td>0</td> <td>1</td> <td>1.4142</td> <td>1.7321</td> <td>2</td> </tr> </table>	x	0	1	2	3	4	y = $\sqrt{x}$	0	1	1.4142	1.7321	2	<b>03</b>			
	x	0	1	2	3	4												
y = $\sqrt{x}$	0	1	1.4142	1.7321	2													
Ans	$\int_a^b f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$ $a = 0, b = 4 \text{ and } h = 1$ $\therefore \int_0^4 \sqrt{x} dx = \frac{1}{2} [(0 + 2) + 2(1 + 1.4142 + 1.7321)]$ $= 5.1463$	2 1																
a)(ii)	Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using trapezoidal rule by using following data: <table border="1" style="margin: 10px auto;"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td>y = <math>\frac{1}{1+x^2}</math></td> <td>1</td> <td>0.5</td> <td>0.2</td> <td>0.1</td> <td>0.588</td> <td>0.0385</td> <td>0.027</td> </tr> </table>	x	0	1	2	3	4	5	6	y = $\frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.588	0.0385	0.027	<b>03</b>
x	0	1	2	3	4	5	6											
y = $\frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.588	0.0385	0.027											
Ans	$\int_a^b f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$ $a = 0, b = 6 \text{ and } h = 1$ $\therefore \int_0^6 \frac{1}{1+x^2} dx = \frac{1}{2} [(1 + 0.027) + 2(0.5 + 0.2 + 0.1 + 0.588 + 0.0385)]$ $= 1.94$	2 1																
b)	Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by Simpson's 1/3 <sup>rd</sup> rule by taking 6 sub intervals.	<b>06</b>																
Ans	<p>Let <math>y = \frac{1}{1+x^2}</math>    <math>a = 0, b = 1</math> and <math>n = 6</math></p> $\therefore h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$ <table border="1" style="margin: 10px auto;"> <tr> <td>x</td> <td>0</td> <td><math>\frac{1}{6}</math></td> <td><math>\frac{1}{3}</math></td> <td><math>\frac{1}{2}</math></td> <td><math>\frac{2}{3}</math></td> <td><math>\frac{5}{6}</math></td> <td>1</td> </tr> <tr> <td>y = <math>\frac{1}{1+x^2}</math></td> <td>1</td> <td><math>\frac{36}{37}</math></td> <td><math>\frac{9}{10}</math></td> <td><math>\frac{4}{5}</math></td> <td><math>\frac{9}{13}</math></td> <td><math>\frac{36}{61}</math></td> <td><math>\frac{1}{2}</math></td> </tr> </table> <p>Using Simpson's 1/3<sup>rd</sup> rule</p> $\int_a^b f(x) dx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$	x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1	y = $\frac{1}{1+x^2}$	1	$\frac{36}{37}$	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{9}{13}$	$\frac{36}{61}$	$\frac{1}{2}$	1 2
x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1											
y = $\frac{1}{1+x^2}$	1	$\frac{36}{37}$	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{9}{13}$	$\frac{36}{61}$	$\frac{1}{2}$											

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6.	b)	$\therefore \int_0^1 f(x) dx = \frac{1}{3} \left[ \left(1 + \frac{1}{2}\right) + 4 \left(\frac{36}{37} + \frac{4}{5} + \frac{36}{61}\right) + 2 \left(\frac{9}{10} + \frac{9}{13}\right) \right]$	2																
		$\therefore \int_0^1 \frac{1}{1+x^2} dx = 0.7854$	1																
		<b>OR</b>																	
		Let $y = \frac{1}{1+x^2}$ $a = 0, b = 1$ and $n = 6$																	
		$\therefore h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6} = 0.1667$	1																
		<table border="1"> <tr> <td><math>x</math></td> <td>0</td> <td>0.1667</td> <td>0.3334</td> <td>0.5001</td> <td>0.6668</td> <td>0.8335</td> <td>1</td> </tr> <tr> <td><math>y = \frac{1}{1+x^2}</math></td> <td>1</td> <td>0.9730</td> <td>0.9</td> <td>0.8</td> <td>0.6922</td> <td>0.5901</td> <td>0.5</td> </tr> </table>	$x$	0	0.1667	0.3334	0.5001	0.6668	0.8335	1	$y = \frac{1}{1+x^2}$	1	0.9730	0.9	0.8	0.6922	0.5901	0.5	2
	$x$	0	0.1667	0.3334	0.5001	0.6668	0.8335	1											
	$y = \frac{1}{1+x^2}$	1	0.9730	0.9	0.8	0.6922	0.5901	0.5											
		Using Simpson's $1/3^{rd}$ rule																	
		$\int_a^b f(x) dx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$																	
	$\therefore \int_0^1 f(x) dx = \frac{0.1667}{3} [(1+0.5) + 4(0.9730 + 0.8 + 0.5901) + 2(0.9 + 0.6922)]$	2																	
	$\int_0^1 \frac{1}{1+x^2} dx = 0.7855$	1																	
	-----																		
	c)	Using Simpson's $3/8^{th}$ rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates.	06																
	Ans	Here $n = 6$																	
		$y = e^{-x^2}$ $a = 0, b = 0.6$																	
		$\therefore h = \frac{b-a}{n} = \frac{0.6-0}{6} = 0.1$	1																
		<table border="1"> <tr> <td><math>x</math></td> <td>0</td> <td>0.1</td> <td>0.2</td> <td>0.3</td> <td>0.4</td> <td>0.5</td> <td>0.6</td> </tr> <tr> <td><math>y = e^{-x^2}</math></td> <td>1</td> <td>0.99</td> <td>0.9608</td> <td>0.9139</td> <td>0.8521</td> <td>0.7788</td> <td>0.6977</td> </tr> </table>	$x$	0	0.1	0.2	0.3	0.4	0.5	0.6	$y = e^{-x^2}$	1	0.99	0.9608	0.9139	0.8521	0.7788	0.6977	2
$x$	0	0.1	0.2	0.3	0.4	0.5	0.6												
$y = e^{-x^2}$	1	0.99	0.9608	0.9139	0.8521	0.7788	0.6977												
		Using Simpson's $3/8^{th}$ rule.																	
		$\int_a^b f(x) dx = \frac{3h}{8} [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + \dots + y_{n-1}) + 2(y_3 + y_6 + \dots + y_{n-3})]$																	
		$\therefore \int_0^{0.6} e^{-x^2} dx = \frac{3(0.1)}{8} [(1+0.6977) + 3(0.99 + 0.9608 + 0.8521 + 0.7788) + 2(0.9139)]$	2																
		$\therefore \int_0^{0.6} e^{-x^2} dx = 0.5351$	1																

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		<p><b><u>Important Note</u></b> <i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p>	

Pinnacle